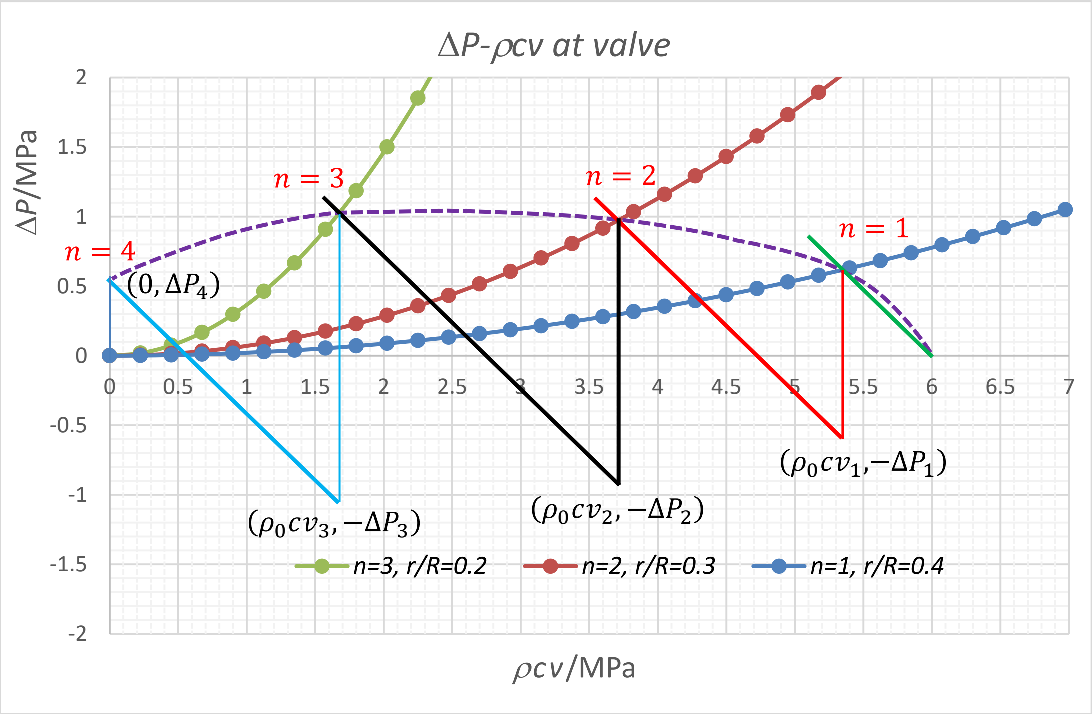
[Marking Scheme] *Theoretical Question 1*

Water Hammer

total	(Task) points	Marking Scheme for Answers to the Problem
Part A 2.2	(A.1) 1.6	*Excess pressure of pressure wave* $\alpha = -(1 + v_0/c)$ ➢ 0.1 expression for impulse. ➢ 0.1 expression for momentum change. ➢ 0.1 equating impulse to momentum change ➢ 0.2 correct equation of continuity for compressible fluid. ➢ {0.1 solving by use of energy conservation} ➢ 0.2 negative sign of α ➢ 0.3 correct magnitude $\lvert\alpha\rvert = 1 + v_0/c$ ➢ {0.1 for $\lvert\alpha\rvert \approx 1$} *Speed of propagation* $\beta = -v_0,\ \gamma = 1 \approx (1 + \Delta P_s/B)$ ➢ 0.1 realizing $-\Delta V/V_0 = \Delta\rho/\rho_1 \approx \Delta\rho/\rho_0$ ➢ 0.1 negative sign of β ➢ 0.2 correct magnitude $\lvert\beta\rvert = v_0$ ➢ 0.2 $\gamma = 1 \approx (1 + \Delta P_s/B)$ ➢ {0.1 if $]\gamma[\approx 1$}
	(A.2) 0.6	*Numerical values of c and ΔP_s for water flow.* ➢ 0.2 + 0.1 for magnitude and unit of $c = 1.5 \times 10^3\ \mathrm{m/s}$. ➢ 0.2 + 0.1 for magnitude and unit of $\Delta P_s = 5.9\ \mathrm{MPa}$. ➢ {0.1 + 0.1 for correct order of magnitude for c and ΔP_s}
Part B 1.0	(B.1) 1.0	*Excess Pressure at valve inlet.* $\Delta P_{\mathrm{in}} = \frac{k}{2}\rho_0 v_{\mathrm{in}}^2,\ k = \left[\frac{1}{C_c^2}\left(\frac{R}{r}\right)^4 - 1\right]$ ➢ 0.2 using inlet and vena contracta in Bernoulli theorem. ➢ 0.1 correct equation of continuity for incompressible fluid ➢ 0.1 deduce $r_c^2 = r^2 C_c$. ➢ 0.1 deduce $v_c = \frac{1}{C_c}\left(\frac{R}{r}\right)^2 v_{\mathrm{in}}$. ➢ 0.5 obtain $\Delta P_{\mathrm{in}} = \frac{k}{2}\rho_0 v_{\mathrm{in}}^2$ with correct k. ➢ {0.2 for $\Delta P_{\mathrm{in}} \propto v_{\mathrm{in}}^2$.}
Part C 1.8	(C.1) 0.6	*Pressure and velocity when valve fully open.* $P_0 = P_a\ \ v_0 = \sqrt{2gh}$ ➢ 0.1 correct equation of Bernoulli theorem. ➢ 0.1 correct equation of continuity. ➢ 0.1 realizing $C_c(r = R) = 1.0$ ➢ 0.1 $v_0 = \sqrt{2gh}$ ➢ 0.2 $P_0 = P_a$.
	(C.2) 1.2	*Pressure $P(t)$ and flow velocity $v(t)$ as $t \to \tau/2$ and $t \to \tau$.* ➢ 0.3 for $P(\to \tau/2) = P_0 + \rho_0 c v_0$ ➢ {0.1 for $P(\to \tau/2) = \rho_0 c v_0$} ➢ 0.3 for $v(\to \tau/2) = 0$ ➢ 0.3 for $P(\to \tau) = P_0 + \rho_0 g h = P_h$ ➢ {0.1 for $P(\to \tau) = P_0$} ➢ 0.3 for $v(\to \tau) = -v_0 + gh/c$ ➢ {0.1 for $v(\to \tau) = -v_0$}

Part D $\boxed{5.0}$	(D.1) 3.0	*Recursion relations for* ΔP_n *and* v_n. $\frac{\Delta P_n}{\rho_0 c} = -(v_n - v_{n-1}) - \frac{\Delta P_{n-1}}{\rho_0 c}$ $(n = 1,2,3,4)$ $\frac{v_n}{c} = \frac{-1+\sqrt{1+2k_n\left(\frac{v_{n-1}}{c}-\frac{\Delta P_{n-1}}{\rho c^2}\right)}}{k_n}$ $(n = 1,2,3)$ ➢ 0.2 setting $h = 0$ to simplify equations. ➢ 0.2 use $\Delta P = \mp\rho_0 c\Delta v$ for waves moving in $\mp x$ direction. ➢ 0.2 sign change of ΔP upon reflection at reservoir end. ➢ 0.2 no sign change of Δv upon reflection at reservoir end. ➢ 0.2 no sign change of ΔP upon reflection at valve end. ➢ 0.2 sign change of Δv upon reflection at valve end. ➢ 1.0 correct recursion formula for ΔP_n, n = 1,2,3,4. ➢ 0.4 use $\Delta P_n = \frac{1}{2}k_n\rho_0 v_n^2$ to eliminate ΔP_n in recursion formula ➢ 0.2 take positive root when solving for $\frac{v_n}{c}, n = 1,2,3$
	(D.2) 2.0	ΔP_n *and* $\rho_0 c v_n$ *by graphical method.* $\frac{\Delta P_n}{\rho_0 c} = -(v_n - v_{n-1}) - \frac{\Delta P_{n-1}}{\rho_0 c}$ ➢ 0.4 (0.1 each) ΔP_n vs. $\rho_0 c v_n$ line $(n = 1,2,3,4)$ passing through $(\rho_0 c v_{n-1}, -\Delta P_{n-1})$ with slope = -1 $(n = 1,2,3,4)$. ➢ 0.3 (0.1 each) parabola for ΔP_n vs. v_n curve $(n = 1,2,3)$. ➢ 0.1 Start with $(\rho_0 c v_0 = 6.0\ \text{MPa}, \Delta P_0 = 0)$ ➢ 0.1 End with $v_4 = 0$ ➢ 0.4 (0.1 each) each label n at $(\rho_0 c v_n, \Delta P_n)$ $(n = 1,2,3,4)$ ➢ 0.4 (0.1 each) estimate of ΔP_n $(n = 1,2,3,4)$. ➢ 0.3 (0.1 each) each estimate of $\rho_0 c v_n$ $(n = 1,2,3)$ Refer to plot and table on next page for values of $(\rho_0 c v_n, \Delta P_n)$.

Partial outcomes obtained for later problems which are incorrect solely because of errors being carried forward but are otherwise reasonable will not be further penalized. However, this rule does not apply to incorrect final outcomes.



Excess Pressures and particle velocities at the valve for slow closing

n	r_n/R	C_n	k_n	$v_n/(\mathrm{m/s})$	$\rho_0 c v_n/\mathrm{MPa}$	$\Delta P_n/(\mathrm{MPa})$	$\Delta P_n/(\rho_0 c v_0)$
0	1.00	1.00	0.0	4.0	6.0	0.0	0.0
1	0.40	0.631	97.1	3.6	5.8	0.62	10 %
2	0.30	0.622	318.	2.5	3.8	1.0	17 %
3	0.20	0.616	1646.	1.1	1.7	1.1	18 %
4	0.00			0.0	0.0	0.64	11 %

$\rho_0 c = 1.50 \times 10^6\ \mathrm{kg\ m^{-2}\ s^{-1}}$ $\qquad v_0 = 4.0\ \mathrm{m/s}$