Theory 3: Magnetic Levitation – Marking Scheme

Part A Sudden appearance of a magnetic monopole (3.0 points)

Initial response **(1.6 points)**

A.1	$\vec{B}_{\rm mp} = \frac{\mu_0 q_{\rm m}}{4\pi}\frac{(z-h)\hat{z}+\vec{\rho}}{[(z-h)^2+\rho^2]^{3/2}}$	**0.1**	**0.4**
	$\vec{B}' = \frac{\mu_0 q_{\rm m}}{4\pi}\frac{(z+h)\hat{z}+\vec{\rho}}{[(z+h)^2+\rho^2]^{3/2}}$	**0.2**	
	$\vec{B} = \frac{\mu_0 q_{\rm m}}{4\pi}\left[\frac{(z-h)\hat{z}+\vec{\rho}}{[(z-h)^2+\rho^2]^{3/2}} + \frac{(z+h)\hat{z}+\vec{\rho}}{[(z+h)^2+\rho^2]^{3/2}}\right]$	**0.1**	
A.2	$\vec{B} = 0$	**0.2**	**0.2**
A.3	$B_z' = 0$ at $z = 0$	**0.1**	**0.4**
	$\Phi_{\rm B} = 0$ at $z = 0$	**0.1**	
	$B_z' = 0$ at $z = -d$	**0.1**	
	$\Phi_{\rm B} = 0$ at $z = -d$	**0.1**	
A.4	$B_\rho(\rho, z=0) d\rho = \mu_0\, j(\rho)\, d\rho \cdot d$	**0.4**	**0.6**
	$\vec{j}(\vec{\rho}) = \frac{1}{\mu_0 d}\hat{z}\times\vec{B}(\vec{\rho}, z=0) = \frac{q_{\rm m}}{2\pi d}\frac{\hat{z}\times\vec{\rho}}{(h^2+\rho^2)^{3/2}}$	**0.2**	

Subsequent response **(1.4 points)**

A.5	$\left.\frac{\partial B_z'}{\partial z}\right\|_z - \left.\frac{\partial B_z'}{\partial z}\right\|_{-d-z} = \mu_0\sigma(d+2z)\frac{\partial B_z'}{\partial t} \approx \mu_0\sigma d\frac{\partial B_z'}{\partial t}$	**0.2**	**0.6**
	$\left.\frac{\partial B_z'}{\partial z}\right\|_z = -\left.\frac{\partial B_z'}{\partial z}\right\|_{-d-z}$	**0.2**	
	$\frac{\partial}{\partial t}B_z'(\rho, z; t) = 2/(\mu_0\sigma d)\times\frac{\partial}{\partial z}B_z'(\rho, z; t)$	**0.2**	
A.6	$B'_z(\rho, 0; t) = f(\rho, z+v_0 t)$ near $z \approx 0$	**0.4**	**0.4**
A.7	At $t=0$ $B_z'(\rho, z \geq 0)$ is of the form $F(\rho, z+h)$	**0.1**	**0.4**
	For $t > 0$ $z \longrightarrow z + v_0 t$	**0.1**	
	$v_0 = 2/(\mu_0\sigma d)$	**0.2**	

Part B Magnetic force acting on a point-like magnetic dipole moving at a constant *h* with a constant velocity (4.0 points)

A moving monopole **(1.5 points)**

B.1	Present positions of $q_{\rm m}$: $(x, z) = [-nv\tau, -h - nv_0\tau]$, for $n \geq 0$.	**0.4**	**0.8**
	Present positions of $-q_{\rm m}$: $(x, z) = [-(n+1)v\tau, -h - nv_0\tau]$, for $n \geq 0$.	**0.4**	

B.2	Magnetic potential : $\Phi_+(x,z) = \frac{\mu_0 q_m}{4\pi}\left[\sum_{n=0}^{\infty}\frac{1}{\sqrt{(x+nv\tau)^2+(z+h+nv_0\tau)^2}} - \sum_{n=0}^{\infty}\frac{1}{\sqrt{(x+(n+1)v\tau)^2+(z+h+nv_0\tau)^2}}\right]$	**0.3**	**0.7**
	$\Phi_+(x,z) = \frac{\mu_0 q_m}{4\pi\tau}\int_0^{\infty} dt'\left[\frac{1}{\sqrt{(x+vt')^2+(z+h+v_0t')^2}} - \frac{1}{\sqrt{(x+vt'+v\tau)^2+(z+h+v_0t')^2}}\right]$	**0.2**	
	$\Phi_+(x,z) = \frac{\mu_0 q_m v}{4\pi}\frac{1}{(z+h)v - v_0x}\left[\frac{z+h}{\sqrt{x^2+(z+h)^2}} - \frac{v_0}{\sqrt{v^2+v_0^2}}\right]$	**0.2**	

A moving dipole **(1.5 points)**

B.3	$\Phi_\mathrm{T}(x,z) = \Phi_+(x,z) + \Phi_-(x,z)$ where $\Phi_-(x,z) = -\Phi_+(x,z-\delta_\mathrm{m})$	**0.2**	**1.5**
	$\Phi_\mathrm{T}(x,z) = \Phi_+(x,z) - \Phi_+(x,z-\delta_\mathrm{m}) = \delta_\mathrm{m} \times \partial\Phi_+(x,z)/\partial z$	**0.2**	
	$\Phi_\mathrm{T}(x,z) = -\frac{\mu_0 m v}{4\pi}\left[\frac{v}{[(z+h)v-v_0x]^2}\left(\frac{z+h}{\sqrt{x^2+(z+h)^2}} - \frac{v_0}{\sqrt{v^2+v_0^2}}\right) - \frac{x^2}{[(z+h)v-v_0x][x^2+(z+h)^2]^{3/2}}\right]$	**0.3**	
	$F_z = -q_m \frac{d}{dz}\Phi_\mathrm{T}(0,z)\Big\vert_{z=h} + q_m \frac{d}{dz}\Phi_\mathrm{T}(0,z)\Big\vert_{z=h-\delta_\mathrm{m}}$	**0.2**	
	$F_z = \frac{3\mu_0 m^2}{32\pi h^4}\left[1 - \frac{v_0}{\sqrt{v^2+v_0^2}}\right]$	**0.2**	
	$F_x = -q_m \frac{d}{dx}\Phi_\mathrm{T}(x,h)\Big\vert_{x=0} + q_m \frac{d}{dx}\Phi_\mathrm{T}(x,h-\delta_\mathrm{m})\Big\vert_{x=0}$	**0.2**	
	$F_x = -\frac{3\mu_0 m^2}{32\pi h^4}\frac{v_0}{v}\left[1 - \frac{v_0}{\sqrt{v^2+v_0^2}}\right]$	**0.2**	

Relation between v_0 and v **(1.0 points)**

B.4	$v_0 = \frac{2}{\mu_0\sigma d} = \frac{2}{4\pi\times10^{-7}\times5.9\times10^7\times0.5\times10^{-2}} = 5.4\ \mathrm{m/s}$	**0.3**	**0.3**
B.5	In the $v < v_\mathrm{c}$ regime: $v_0(v) = v_0$	**0.1**	**0.4**
	In the $v > v_\mathrm{c}$ regime: $v_0(v) = \frac{2}{\mu_0\sigma\delta} = \frac{2}{\mu_0\sigma}\sqrt{\frac{\omega\mu_0\sigma}{2}}$	**0.1**	
	$\omega = v/h$	**0.1**	
	$v_0(v) = v_0\sqrt{\frac{d}{h}}\sqrt{\frac{v}{v_0}}$	**0.1**	
B.6	$\delta = d$	**0.1**	**0.3**
	$v_\mathrm{c} = \frac{2h}{d^2\mu_0\sigma} = v_0\frac{h}{d}$	**0.2**	

Part C Motion of the magnetic dipole when the conducting thin film is superconducting (3.0 points)

C.1	**Approach 1:** Start from the total magnetic potential		**1.2**
	$$\Phi_{\mathrm{T}}(x,z) = -\frac{\mu_0 q_{\mathrm{m}}}{4\pi}\frac{1}{\sqrt{x^2+(z+h)^2}} + \frac{\mu_0 q_{\mathrm{m}}}{4\pi}\frac{1}{\sqrt{(x-\delta_{\mathrm{m}})^2+(z+h)^2}}$$	**0.3**	
	$$F_z' = (-q_{\mathrm{m}})\left[-\frac{\partial}{\partial z}\Phi_{\mathrm{T}}\right]\Bigg\|_{\substack{x=0,\\ z=h}} + q_{\mathrm{m}}\left[-\frac{\partial}{\partial z}\Phi_{\mathrm{T}}\right]\Bigg\|_{\substack{x=\delta_{\mathrm{m}},\\ z=h}}$$	**0.3**	
	$$F_z' = \frac{3\mu_0 m^2}{64\pi h^4}$$	**0.4**	
	$$h_0 = \left[\frac{3\mu_0 m^2}{64\pi M_0 g}\right]^{\frac{1}{4}}$$	**0.2**	
	Approach 2: Start from the force		
	$$F_z' = 2\frac{\mu_0 q_{\mathrm{m}}^2}{4\pi}\left[\left(\frac{1}{2h}\right)^2 - \frac{2h}{(\delta_m^2+(2h)^2)^{3/2}}\right]$$	**0.6**	
	$$F_z' = \frac{3\mu_0 m^2}{64\pi h^4}$$	**0.4**	
	$$h_0 = \left[\frac{3\mu_0 m^2}{64\pi M_0 g}\right]^{\frac{1}{4}}$$	**0.2**	
C.2	$$\frac{dF_z'}{dz} = -k = -M_0\Omega^2$$	**0.5**	**0.8**
	$$\Omega = \sqrt{\frac{4g}{h_0}}$$	**0.3**	

C.3	$$h_0 = \left[\frac{3\mu_0\left(\frac{4}{3}\pi R^3 M\right)^2}{64\pi\left(\frac{4}{3}\pi R^3\rho_0 g\right)}\right]^{1/4} = \left[\frac{R^3 M^2\mu_0}{16\rho_0 g}\right]^{1/4}$$	**0.3**	**0.7**
	$$h_0 = \left[\frac{10^{-18}\times 75^2\times 10^{-4}}{16\times 7400\times 9.8\times\mu_0}\right]^{1/4}\ \mathrm{m}$$	**0.2**	
	$h_0 = 25\ \mu\mathrm{m}$	**0.2**	
C.4	$$\Omega = \sqrt{\frac{4g}{h_0}} = \sqrt{\frac{4\times 9.8}{30\times 10^{-6}}}\ \mathrm{s}^{-1} = 1.3\ \mathrm{kHz}$$	**0.3**	**0.3**